Annals of Finance **1**, 395–421 (2005)
DOI: 10.1007/s10436-005-0017-8

Annals of Finance

Informational asymmetries and a multiplier effect on price correlation and trading*

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Received: March 3, 2005; revised version: April 27, 2005

Summary. In this paper we show how rational expectations equilibrium models with asymmetric information, without market frictions, can generate extreme co-movements in asset prices. Information asymmetries generate a multiplier effect on price correlation - a World Bank definition of financial contagion. This is shown in two frameworks: perfect and imperfect competition. In the first framework, we also model a version of home-bias, showing why information sharing explains cross-country capital flows. In the second framework, we provide closed form solutions for a model with multiple insiders and assets that generalize the ideas in [10].

Keywords and Phrases: Information asymmetries, Financial contagion, Home-bias.

JEL Classification Numbers: G11, G12, G15.

1 Introduction

Huge correlation in assets prices, that cannot be easily justified by fundamentals, is pervasive during financial crises. For example, if Argentina experiences decreased demand for its financial assets due to lack of sound policies the stock market might plunge and take with it several neighboring markets. That's why financial markets participants, say, in Brazil fear any setbacks in the Argentine economy.[1] So, why are

* I would like to thank C.D. Aliprantis (the editor), Markus Brunnermeier, Haluk Ergin, Ulrich Horst, Deniz Igan, Áureo de Paula, Ailsa Röell, Wei Xiong and an anonymous referee for helpful comments. I am particularly indebeted to José Scheinkman for encouraging me to pursue this topic and for great help on the directions of the work. I gratefully acknowledge financial support from the NSF (grant 0001647).

[1] Recently, end of 2001 beginning of 2002, Argentina has faced a severe crisis emanating, mostly, from its fixed exchange rate regime. This crisis, however, did not contaminate any neighboring markets. Later in the paper we explain why this is the case, and how the present model can explain this instance of immunity.

Brazilian and Argentine asset prices so correlated? It has become common knowledge that these prices, albeit having correlated fundamentals, should not show such strong correlation. There seems to exist a multiplier effect on these markets. The empirical phenomenon is well documented[2], hence deserves a thorough analysis and theoretical justification. Most approaches so far rely on financial constraints as the mechanism that transmits shocks from one country to another. We propose a model that does not need this "market imperfection". We show that these excess co-movements can be justified via a model of asymmetric information. We also justify another empirical finding, namely, "home-bias" in capital flows. Countries that are closely related, sharing cultural ties, seem to have a higher exchange of capital between them than with others.

The Asian crisis is an example of the mentioned phenomenon. On July 2nd, 1997 the peg of the Thailandese currency collapsed, and other Asian markets followed the slippery slope. [6] analyze this event. They empirically test the presence of contagion by testing if the correlations in markets increase significantly during crises. There is evidence of substantial contagion in the foreign debt markets, with a somewhat less striking evidence for stock markets. The *Tequila Effect* on 1994 after the Mexican crisis is another example of contagion between markets, as well as the crisis in Russia in 1998 and in Brazil in 1999 with their aftermath effects spreading to related economies.

In this paper we propose an equilibrium model that endogenously generate *Financial Contagion* and *Home-Bias* in capital flows. The key ingredient is the presence of asymmetries in the information of each market participant. We set up a rational expectations model with asymmetric information and compute the correlation of the asset prices. Then, we compare this with the correlation arising in a symmetric full information equilibrium and with the correlation in the asset payoffs. We propose and justify as a measure of contagion the difference in price correlations. We choose the symmetric full information case as a benchmark since this is the ensuing situation if information is free. We also compare the correlation in returns (payoffs minus prices) in the two equilibria, asymmetric and symmetric information. Again, the results are as expected: in the asymmetric equilibrium asset returns are more correlated than in the symmetric equilibrium.[3,4]

Empirically, crises are usually accompanied by increased noise in financial indicators and asset fundamentals. So, financial crises are closely related to the amount of noise in the information in the market. Even though excess correlation is almost always present in this model, its degree varies. That's why we argue that this model explains financial contagion (excess correlation during crises). We show

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[2] See, for example, [5], [6] and [15].

[3] In previous versions of this paper we would proceed further and endogeneize these asymmetries in the information structure. We approached the problem as a generalization of [24]. However, information acquisition would reflect expenses to participate, get settled, in a market and relied on this interpretation to justify the assumption that, in order to be able to trade, individuals must be informed. So, at the start there would not be any uninformed individuals in this model. This could also be relaxed, as well as allowance for acquiring more than one type of information. We fully omit these results from this version, but they are available from the author upon request.

[4] In addition, it will be shown that in the symmetric equilibrium the correlation in payoffs is higher than the price correlation. This is an important point since we do not want the exacerbated correlation (relative to payoffs) to be the result of the operation of markets, but the result of information asymmetries.

that, as the informational variables (proxy for asset fundamentals) become nosier, the excess correlation phenomenon is exacerbated. And, on the other hand, when information is relatively stable there is almost no excess correlation. So, during crises (high information volatility) financial contagion obtains, and in calmer periods (stable information) there is almost no excess correlation. In summary, we argue that informational asymmetries are more important during crises.

We should stress that the objective of this work is to explain the *exacerbation* in correlation during periods of crisis, *not the appearance* of it where it did not exist. Hence, we take as starting point countries/assets that are inherently correlated, and show how information asymmetries might generate a multiplier effect. This view of Financial Contagion as an exacerbation of correlation is justified by the empirical literature[5] and fits into what the World Bank defines as contagion.[6]

[25] presents evidence of the existence of the aforementioned asymmetries. [7] Analyzing profits of traders in 8 European countries, with equal access to a trading system, he finds significant evidence of underperformance of non-German traders trading in German stocks. Overall, the evidence points to the relevance of culture and geographic proximity as a proxy for information asymmetries. The closer two countries are the closer are their information set. So, nationals seem to have better information than international investors. This evidence fits nicely into our proposed framework, even if information was costly.

The basic framework to be adopted here is the same as in the relevant literature, that started with [21]. The structure is the following: there are 2 periods and 4 assets. There is a risk free asset and 3 risky assets, 2 of which are inherently correlated and one uncorrelated with the rest. We can think of the 2 correlated assets as, for example, Brazilian and Argentine assets and imagine the third asset as a composite international portfolio (not including either of the other two).[8] We assume that there is a continuum of investors with measure 1, each one with a CARA utility function, with some deterministic, and publicly known, initial wealth. The assets' payoffs are jointly normally distributed. Each investor will be informed about one of the assets. However, since two of them are correlated, learning about one entitles that person to information about the other. Independently of the information a trader has, he has full access to the assets, there is no exogenously imposed market segmentation. However, at this stage, we require that the investor be informed to participate in the market. We solve the model considering a given information structure.[9] We

[5] See, for instance, [6].

[6] The World Bank has three basic definitions of contagion: broad, restrictive and very restrictive. The broad definition states that contagion is any cross-country spillover effect, during bad or good times. The restrictive definition calls contagion the transmission of shocks or cross-country correlation beyond any fundamental link. Finally, the very restrictive definition requires the above exacerbation during crisis times relative to tranquil times. This information was obtained at www1.worldbank.org.

[7] Further evidence is discussed in the related literature section.

[8] We concentrate on these 2 assets and take a more or less agnostic view of the rest of the world. This simplification is mostly a computational one, as the number of parameters in the model grows extremely fast with the number of assets. Increasing the number of assets would make the algebra even more cumbersome without adding much to the interpretation of the results.

[9] As mentioned before, we could go back and solve for the learning stage as well, endogeneizing the informational asymmetries. This was done in a previous version of this paper.

assume that the supply of the risky assets is random[10] and multivariate normally distributed.[11]

In the first period each investor is endowed with a piece of information, and, conditional on his information and on the equilibrium price, he submits a demand for the assets. There is no consumption in this period. The random supply is realized and the market clearing condition determines prices. In the second period, payoffs are realized and consumption takes place.

As a further exercise, the market microstructure is changed to check the strength of our results. We consider a case in which markets are not perfectly competitive. To do so, we generalize the work of Kyle in [30]. He considers an imperfectly competitive market with one informed individual, a market maker and noise traders and analyzes the informativeness of prices, among other related issues. We allow for multiple assets and insiders and information asymmetries across insiders. [10] generalizes Kyle's model in similar fashion, however they do not model information asymmetries across insiders, only between insiders and noise traders. So, we are able to provide closed form solutions for a model with multiple insiders and assets that generalize the ideas in [10]. The results about exacerbated price correlation, under asymmetric information, remain true for this new market microstructure reinforcing our results.

The rest of this paper is organized as follows: In Section 2 we briefly review the related literature, putting the present work into perspective, stressing its importance and main differences to existing models of information in financial markets, and, especially, the differences to existing models of financial contagion. In Section 3 the model is described in greater detail. Section 4 solves for the benchmark case of the full information symmetric equilibrium. In Section 5 we describe the asymmetric information case and the solution to the model. In Section 6 we analyze the numerical results obtained. In Section 7 we discuss the modification of the market microstructure and its results on the correlation of asset prices. Section 8 concludes.

2 Related literature

The literature on models of information and financial markets is vast, addressing a variety of economic phenomena. It is not the objective of this paper to present a complete literature review.

[24] addresses the informational content of prices. They consider a model with informed and uninformed individuals, but where the decision to be of one type or another is endogenous.

[4] analyzes financial contagion in a very different setup. Their model is one of complete information, and their focus is on the credit/banking market. [31] proposes contagion as a result of, what they call, a wealth effect. They use a continuous-time model with 3 types of traders: noise traders, long-term investors and convergence

[10] The usual interpretation is that of noise traders, the supply being the negative of their demand.

[11] We need some additional noise in the model, otherwise all information would end up being reflected on the equilibrium price, rendering it worthless. For a discussion of this fact see, among others, [24], [21], [1], [26] and [14].

traders. They allow for 2 risky assets and a risk free one. Convergence traders are fully rational with logarithmic utility function and infinite horizon. Long term investors are not fully rational. With this utility function format we have assets' demand increasing in wealth. This makes trading patterns pro-cyclical. So, when convergence traders lose money they may end up liquidating positions in both markets generating the contagion effect. More specifically, shocks to any asset's return are amplified by a portfolio rebalancing and may be transmitted to the other asset. In a similar study, [20] proposes a model where 2 identical assets are traded in segmented markets. They analyze equilibrium when some investors can invest in only one market, but others, the arbitrageurs, can invest in both. The competitive arbitrageurs will tend to exploit any difference in prices that may arise in equilibrium between the 2 assets. However, these agents are assumed to be financially constrained, their position cannot be greater than a certain multiple of their wealth. They have 2 different constraints, one for each market. The authors are more interested in welfare analysis, but show a similar contagion effect as [31].

The advantages of the present work can be emphasized on a few points. First, we believe that the information asymmetries are more plausible and less restrictive to explain financial contagion and home-bias. When this phenomenon came to the spotlight (especially after the Tequila Crisis and its subsequent effects), it was considered a somewhat puzzling observation. It could not be easily explained within the traditional economics and finance framework. Efficient frictionless markets and fully rational individuals could not coexist with the phenomenon. Several studies took, then, alternative routes: market frictions and behavioral models. One of the main intents of this work is to reconcile financial contagion (and home bias) with a fully rational, frictionless market model. To fulfill this goal, informational asymmetries seem to be the key. The above mentioned works have a few restrictive assumptions. For example, in [20] some investors do not have access to all markets. We allow access to all assets. In [31] not all investors are fully rational and some investors' preferences are not modeled. Here all investors are fully rational and have well defined preferences. In both papers financial constraints play a role directly ([20]) or indirectly ([31]).

Several empirical studies[12] analyze the existence of asymmetric information in international markets. The findings suggest that local investors exit the markets before international investors and that in general the evidence points to the existence of asymmetries in favor of nationals, i.e. national (local) investors seem to have better information about the local assets. This evidence reinforces our view of the relevance of informational asymmetries.

As a further contribution of this work we point out to its ability to explain another phenomenon. As will be seen later in the paper, investors will trade more heavily on assets about which they have information. Presumably, these assets are the ones that they perceive as local, i.e., assets that are based on their home country and/or on countries that are intimately related to their own.[13] In this sense, we can relate the present paper to [18] and [32]. The first paper reports on the empirical

[12] See for instance: [16], [17], [12], [9], [28] and [27].

[13] In the interpretation provided in the paper, the assets are Brazilian and Argentine securities. Therefore, the interpretation is clear, since Brazil and Argentina are neighbors and closely related.

findings of home bias. Investors tend to hold a highly concentrated portfolio, not using the international market to diversify away risk. They conclude that investors must expect the returns on their domestic markets to be several hundred basis points above returns on other markets. The present model could be used to reinterpret the facts, justifying a sort of modified home bias. If we think about the assets sharing a common factor as being traded in close-by countries, in the same region, we would have a model that says that investors trade more heavily on assets that are geographically close to where they live. We call this a modified home bias since we are not talking about a country, but a continent [14], and we cannot say that the investors are holding more of the home assets, only that they trade more aggressively on them.[15] The advantage of this interpretation is that investors do not have to be making systematic mistakes and expect abnormal returns from their home market. The home-bias is a result of fully rational agents that, endogenously, and optimally, decide to posses asymmetric information. This theoretical formulation is further justified by the empirical work of Portes and Rey in [32]. They show that geographical proximity is a major determinant of transaction flows between countries, showing, more interestingly, that distance is a proxy for information asymmetries. Countries that share strong cultural ties have a higher exchange of funds among themselves. Hence, the present model provides a sound theoretical justification for their empirical findings.

The closest paper to this one is [29].[16] It has basically the same starting framework and objectives. However, the definition of contagion is rather different. Here, we take contagion as a multiplier effect. The resulting correlation in an asymmetric equilibrium is higher than its counterpart in the symmetric equilibrium, and also higher than what could be justified by correlated fundamentals. So, it is an analysis of the impact of the market microstructure on the correlation of asset prices. Their work, on the other hand, view contagion as any cross price effect, i.e., a shock to one country being carried through to another. This effect can, however, be shown to exist in a symmetric full information equilibrium, as well. Their model serves best (but not only) to explain, for example, contagion from Asia to Latin America, and not within Latin America. The information asymmetries that they propose are of a different sort. In their model agents are either informed, in which case they have information about all assets in the world, or completely uninformed. We model information asymmetries based on its interpretation as a geographical/cultural phenomenon, agents are informed about their "home region". Being in Brazil, makes it easier, culturally and geographically, to know about Brazilian as well as Argentine assets, given that they do have correlated fundamentals (as we model). Our interpretation of information asymmetries is substantiated empirically by several studies. For example, [25] finds evidence that German traders are more informed about

[14] This could easily be changed if we thought about the assets with common factor as being traded in the same country. However, on doing this we would lose the interpretation of international financial contagion. Hence, we restrict ourselves to the previous interpretation, this also helps understand the empirical findings of [32].

[15] On the presence of good news they will be holding more of the home assets, however, given bad news they might even short-sell these assets.

[16] This paper was developed independently of Kodres and Pritsker's. Unfortunately, I was unaware of their paper until it was accepted for publication.

German stocks than foreigners. On the other hand, there is hardly any evidence of players that are globally better informed. There is no evidence of traders that consistently outperform the market (what would be implied by their formulation). Our story is one of specialization in the information market. The model in [29] cannot explain the empirical phenomenon described in [32] since, empirically, the material information asymmetries are across countries and not across individuals.[17] Also, in our imperfect competition framework (see Section 7) we get results that enable us to explain the contagion effects from Asia to Latin America as well as the contagion within Latin America. In this alternative framework all we need is an ex-ante correlation bounded away from zero, even if a very small one. In this sense, the present work is able to "replicate" their results and add to it. Finally, we also analyze exacerbated return correlation, which is not done in [29].

In brief, our model generates financial contagion and a "modified" home-bias with a minimal set of ad-hoc assumptions. In particular, wealth plays no role in the results and markets are as close to perfect as possible: there are no financial constraints, no restrictions on short-selling and traders have access to all markets. The driving force behind our results is the difference in agents' information. This is an appealing assumption both empirically, given the above-mentioned evidence, and theoretically, given that we can endogenize the information structure showing that differential information is part of a general equilibrium.[18]

3 The perfect competition model

In this model there is a continuum of agents with measure 1 and 2 periods. Agents trade in the first period and consume in the second. Each agent is endowed with initial wealth w_0. The initial wealth is allocated to the purchase of assets. There are 4 assets available: 3 risky and 1 risk-free. The assets pay off in the second period, when agents consume. Each agent has the following CARA utility function over final wealth:

$$u(w_1) = -\exp(-\rho w_1). \tag{1}$$

The first asset is the risk free asset, it has a, normalized, price of one and pays off R units in the second period. Let the price of the risky assets be P, a 3 dimensional vector. We assume that each asset pays off X_i for $i \in \{1,2,3\}$, where:

$$X_1 = I_1 + \varepsilon_1, \quad X_2 = I_2 + \varepsilon_2, \quad X_3 = I_1 + \varepsilon_3,$$

or, more compactly, in vector notation: $X = I + \varepsilon$. Assume that $I_i's$ and $\varepsilon_i's$ are jointly normally distributed zero-mean random variables with the following variance-covariance matrices:

[17] What matters is that neighboring countries/markets share common informational variables.
[18] Once more we remind the reader that this extension is omitted but is available from the author.

$$\Omega = \begin{bmatrix} \varpi & 0 \\ 0 & \varpi \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}, \qquad (2)$$

respectively, and $Cov(I_i, \varepsilon_j) = 0\ \forall i, j$. It is clear that asset 2 has payoff completely independent of 1 and 3. On the other hand, assets 1 and 3 have correlated fundamentals. We can view assets 1 and 3 as the Brazilian and Argentine stock markets, for instance, and asset 2 as a composite international (excluding Brazil and Argentina) portfolio. Given this specification we can calculate the variance of X to be equal to $\Omega_2 + \Sigma$, where we define Ω_2 to be:

$$\Omega_2 \equiv \begin{bmatrix} \varpi & 0 & \varpi \\ 0 & \varpi & 0 \\ \varpi & 0 & \varpi \end{bmatrix} \qquad (3)$$

The supply of this assets will be denoted by $U \in \mathbb{R}^3$, a vector valued random variable with a mean zero multinormal distribution and variance-covariance matrix given by:[19]

$$V = \begin{bmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_1 \end{bmatrix}. \qquad (4)$$

We endow the investors with one of two pieces of information, they can learn either I_1 or I_2 in period one. The ε is unobservable, as is U. By learning I_1, a trader has information about assets 1 and 3. There are no uninformed agents in the market, and we assume that a proportion λ of the investors learn about I_1 (let's call this Type 1 investors) and the rest about I_2 (Type 2).[20]

Timing: The timing of the model is as follows: First, agents receive their non-random endowment, then each agent observes his information. Type 1 observes I_1 while Type 2 observes I_2. Then, conditional on their information, traders submit whole demand schedules, so that we are allowing them to condition on prices as well. This is important, since prices reflect private information, and the rational expectations hypothesis (discussed below) makes this notion operational. Then, finally, market clearing determines the price.

Let $D_i \in \mathbb{R}^3$, $i = 1, 2$ be the investors demand for the risky assets, and F_i their demand for the risk-free asset. Investors maximize their expected utility conditional on their information set and subject to their budget constraint:

$$\max_{D_i, F_i} E\left[u(w_{i1}) | I_i, P\right]\ s.t\ w_{i0} = D_i^T \cdot P + F_i. \qquad (5)$$

Each type's final wealth is, then, given by:

$$w_{i1} = \left(w_{i0} - D_i^T \cdot P\right) R + D_i^T \cdot X. \qquad (6)$$

[19] The assumption that the supply variance in markets 1 and 3 is the same is not needed here, however it does simplify the algebra.

[20] One could pose the question of existence of an equilibrium in which a proportion of the population learn about one information and the rest about the other. As mentioned before, we can easily endogenize this information asymmetry.

If we assume that the price has normal distribution (as we later confirm to be the case), then this program collapses to

$$\max_{D_i} E[w_{i1}|I_i,P] - \frac{1}{2}\rho Var[w_{i1}|I_i,P], \tag{7}$$

with optimal demand for the risky asset given by

$$D_i^*(I_i,P) = \frac{(Var[X|I_i,P])^{-1}\cdot(E[X|I_i,P]-PR)}{\rho}. \tag{8}$$

Notice that our derivation of the optimal demand functions depends on the "assumption" that the price vector is multinormally distributed. Therefore, one must concentrate on linear equilibria (linear price functions) to guarantee such property. This is a standard procedure, and one that is followed here, as well.

The following is a standard definition of rational expectations equilibrium.

Definition: *A* **Rational Expectations Equilibrium** *for the security market is a price vector P^* (measurable with respect to (I_1,I_2,U_1,U_2,U_3)) and demand functions $D_i^*(I_i,P^*)$ such that: $D_i^*(I_i,P^*) = \arg\max_{D_i} E[w_{i1}|I_i,P^*] - \frac{1}{2}\rho Var[w_{i1}|I_i,P^*]$ and the market clears,*

$$\lambda D_1^*(I_1,P^*) + (1-\lambda)D_2^*(I_2,P^*) = U.$$

In the full information case the same derivation and definition apply, with the information set of all investors including both I_1 and I_2, being unnecessary to further condition on prices.

4 Symmetric information equilibrium

In this section we describe a benchmark case in which it is assumed that all the investors know both I_1 and I_2. This seems to be the relevant benchmark since, if information was freely available and accessible to all, this would be the ensuing situation. In this case their optimal demand is given by:

$$D^*(I_1,I_2) = (\rho Var[X|I_1,I_2])^{-1}\cdot[E[X|I_1,I_2]-PR], \tag{9}$$

where we can easily calculate $Var[X|I_1,I_2] = \Sigma$ and $E[X|I_1,I_2] = \begin{bmatrix} I_1 & I_2 & I_1 \end{bmatrix}^T$.

We conjecture the following as the functional form of the price vector, restricting attention to linear equilibria:

$$P = \alpha + \beta * \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - \gamma * U, \tag{10}$$

where:

$$\alpha = \begin{bmatrix} a_1 \\ a_2 \\ a_1 \end{bmatrix}, \beta = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \text{ and } \gamma = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}. \tag{11}$$

We then use a market clearing condition to calculate the price as a function of these parameters and the exogenous ones. Then, we equate coefficients of this price with the ones conjectured and solve for α, β and γ. We have, then, the following proposition. All omitted proofs are in the appendix.

Proposition 1. *There exists a unique linear equilibrium price vector with*

$$\alpha = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \beta = \begin{bmatrix} \frac{1}{R} & 0 \\ 0 & \frac{1}{R} \\ \frac{1}{R} & 0 \end{bmatrix} \text{ and } \gamma = \begin{bmatrix} \frac{\rho\sigma^2}{R} & 0 & 0 \\ 0 & \frac{\rho\sigma^2}{R} & 0 \\ 0 & 0 & \frac{\rho\sigma^2}{R} \end{bmatrix}, \tag{12}$$

so that our price vector is normally distributed.

Our main interest is to compare the equilibrium price correlation between assets 1 and 3, $Corr(P_1, P_3) \equiv \frac{Cov(P_1,P_3)}{Std(P_1)Std(P_3)}$. We could specialize this formula given the equilibrium parameters derived above, however, the asymmetric case will not be solved analytically. Therefore, there is no gain in having an analytical expression for the correlation of asset prices in the symmetric case. Similarly, we have that the ex-ante[21] correlation of assets' payoffs is $Corr(X_1, X_3) = \frac{\varpi}{\varpi+\sigma^2}$.

We use the difference in price correlation as our measure of contagion, [22] thinking about tranquil times as periods in which informational asymmetries are not very important. We take tranquility to mean that there is full symmetric information, or something that could be approximated by this (e.g., low variance of the information variables). We show how information asymmetries amplify correlations and view this as a multiplier effect. So we compare price correlation in the symmetric case with the asymmetric case arguing that the first represent normal times while the latter captures the idea of crisis to conclude that correlations are exacerbated during crisis (periods of very volatile fundamentals/information where information asymmetries are important)[23].

5 Asymmetric information equilibrium

Using the optimal demands derived in Section 3 and considering the same conjectured price function as in the previous section we can begin deriving the asymmetric information equilibrium. First, we need the joint distribution of $\Psi \equiv (X, P, I_1, I_2)^T \in \mathbb{R}^8$. This distribution can easily be seen to be multinormal with mean:

$$\begin{bmatrix} 0 & 0 & 0 & \alpha & 0 & 0 \end{bmatrix}^T, \tag{13}$$

[21] By ex-ante we mean before any information is revealed, simple unconditional moments.

[22] Empirically this is the measure used by [6], among others.

[23] It is important to notice that, in the limit, if information variables are deterministic, then the distinction between the symmetric and asymmetric equilibrium disappears.

and variance:

$$\begin{bmatrix} \Omega_2+\Sigma & \begin{bmatrix} \Omega \\ \varpi \; 0 \end{bmatrix}\beta^T & \begin{bmatrix} \Omega \\ \varpi \; 0 \end{bmatrix} \\ \beta\begin{bmatrix} \Omega & \begin{matrix}\varpi \\ 0\end{matrix} \end{bmatrix} & \beta\Omega\beta^T+\gamma V\gamma^T & \beta\Omega \\ \begin{bmatrix} \Omega & \begin{matrix}\varpi \\ 0\end{matrix} \end{bmatrix} & \Omega\beta^T & \Omega \end{bmatrix}. \tag{14}$$

Given this joint distributions we can calculate the conditional moments $Var[X|I_i,P]$ and $E[X|I_i,P]$.[24] The expressions are, however, cumbersome[25], and stating them here would not add any insights. In the proof of the following proposition we flash these expressions out and explore their properties. But, for now, just define: $V_i \overset{d}{=} Var[X|I_i,P],\ E_i \overset{d}{=} E[X|I_i,P]$. We have the following proposition where again we restrict the equilibrium to be linear.

Proposition 2. *There exists a Linear Rational Expectations Equilibrium with the parameters α, β and γ determined by solving*

$$N^{-1}Xz_1=\left[b_{11}\ b_{21}\ b_{31}\right]^T;\ N^{-1}Tz_2=\left[b_{12}\ b_{22}\ b_{32}\right]^T; N^{-1}\frac{T}{(1-\lambda)}V_2\rho=\gamma \tag{15}$$

and $\alpha=\overrightarrow{0}$, where N, T, X, z_1, z_2, x_1 and x_2 depend on endogenous parameters and are depicted in the proof.

This is the system of equations that we use to run our numerical exercises. As it is shown in the proof, in order to arrive at such system we did not fully use the format of our set-up, e.g. the inherent symmetry between asset 1 and 3. In the proof, we further characterize the equilibrium using all the properties of our model. Essentially, the difference is that these equations allow for the equilibrium to be asymmetric, so in our numerical results we allow for this possibility. However, simulations show that invariably the equilibrium is indeed symmetric. So, to dig deeper into the characterization of the equilibrium we impose this structure ex-ante. We believe this is without loss of generality, as explained in the proof and supported by the numerical results.

6 Numerical solution and comparative statics

In this section we briefly describe the results obtained numerically, concentrating on the comparison of asset prices' correlation and on the explanation of home-bias. It was a recurrent fact that some parameter values were always zero, in particular:

[24] See [13].

[25] Remember, though, that, different than in the full information case, these expressions are functions of the parameters of the conjectured price, so we will need them when time comes to equate coefficients. As will be seen, we will be searching for a fixed point of a function $f(.)$, $f: R^{18} \rightarrow R^{18}$.

Table 1. An example of asymmetric equilibrium. The equilibrium parameters are calculated using the following parameters' values: $v_1 = v_2 = 50$, $\rho = 1.5$, $R = 1$, $\sigma^2 = 2$. All parameters that were equal to zero were omitted from this table

ϖ	b_{11}	b_{22}	b_{31}	g_{11}	g_{13}	g_{22}	g_{31}	g_{33}
0.5	60	45.01	60	276.01	84.01	270.04	84.01	276.01
1	95.96	60	95.96	383.89	191.89	360.03	191.89	383.89
1.5	139.85	76.99	139.85	515.56	323.56	461.95	323.56	515.56
2	191.65	95.96	191.65	670.94	478.94	575.79	478.94	670.94

$b_{12} = b_{21} = b_{32} = g_{12} = g_{21} = g_{23} = g_{32} = 0$. The explanation is that the information about asset 2 has no impact on assets 1 and 3 (and vice-versa), also the supply of asset 2 has no impact on assets 1 and 3 (and vice-versa). Another point worth mentioning is the symmetry in assets 1 and 3, as expected by our assumptions: $b_{11} = b_{31}, g_{11} = g_{33}$ and $g_{13} = g_{31}$. We present below a sample of the results (omitting the zeroes):
Note that the only reason the prices of asset 1 and 3 are not the same is the fact that they have different liquidity shocks. The demand schedule of the traders informed about I_1 for these assets is always the same.

6.1 Analysis of the contagion effect

We start now the analysis of excess correlation in prices and returns.

6.1.1 Comparing price and payoff correlations

In the Introduction it was said that exacerbated correlation would be most pervasive in times of crisis. Crisis is viewed here as any situation where the liquidity of the market is really volatile, and, more importantly, information is noisy (information asymmetries are more important). So, whenever there is enough noise in the market, we consider these, times of crisis.[26,27]

We run several numerical examples to investigate the effects of some parameters on the difference in correlation (between the asymmetric and symmetric case) all of then reinforce our intuition about the importance of the information structure.

As we increase the risk aversion coefficient the difference in correlation increases, the same is true for increases in the volatility of supply and in the volatility of the informational variables. These results are intuitive since they have to do with the importance of information. Higher risk-aversion leads to a higher importance of information since being informed reduces risk. Similarly for higher volatility of information variables: Knowing the realization of a very volatile random variable

[26] The defining point of a crisis is considered, here, to be the volatility of fundamentals through information variables.

[27] It is worth pointing out that we used correlation, and not covariance, as a proposed measure of contagion because the former is unit free. I.e., if we multiply two random variables by a constant they still have the same correlation, but different covariances.

Table 2. Excess correlation. We analyze the excess correlation phenomenon here by using as a benchmark the following parameter values: $v_1 = v_2 = 50$, $\rho = 1.5$, $R = 1$, $\sigma^2 = 2$. We vary the volatility of the informational variable to analyze its effect on financial contagion. We make use of the following definitions:

$$D_1 \stackrel{d}{=} Corr_{Asym.\ Eqm}(P_1, P_3) - Corr(X_1, X_3);$$
$$D_2 \stackrel{d}{=} Corr_{Asym.\ Eqm}(P_1, P_3) - Corr_{Sym.\ Eqm}(P_1, P_3);$$
$$D_3 \stackrel{d}{=} Corr_{Sym.\ Eqm}(P_1, P_3) - Corr(X_1, X_3)$$

ϖ	D_1	D_2	D_3
0.5	0.36	0.56	−0.20
1.0	0.47	0.80	−0.33
1.5	0.47	0.90	−0.43
2.0	0.45	0.94	−0.49

helps reduce/control risk. Finally, as the volatility of supply increases the public signal (price) becomes less informative; hence the relative value of the private signal (information variables) increases. All these effects lead to an increase in the importance of asymmetries in the information structure accentuating the difference in correlation. Finally, if we increase the volatility of the individual noise terms (ε) the difference in correlation is reduced. Higher idiosyncractic volatility leads less informative private signals. Therefore, these signals become less important and the differences between the symmetric and asymmetric case tends to diminish. In the limit, as noise becomes infinitely volatile you cannot learn anything about payoffs by observing a signal, hence this difference vanishes. Table 2 provides one such numerical result - the most important one - on excess correlation where we vary the volatility of the information variable. Several other experiments were done but are omitted as a matter of space. The results are all qualitatively the same.

The basic intuition behind the results is as follows. In a symmetric equilibrium the optimal portfolios have the same loadings on each asset. However, in the asymmetric setting agents end up trading more heavily on the assets that they have information about. This gets transported to the price, resulting in prices of those assets being more sensitive to the information variables. Since 2 assets share the same information variable they end up being highly correlated. When news arrive in one market they are also news about the other market, therefore anything affecting the price in one market will cause a movement in the same direction in the other market. And, the market and its information asymmetries act as a multiplier. Put somewhat differently, in this model investors end up "specializing" (concentrating their trades) in some stocks, namely, the ones they have information about.[28] As these investors get different signals (information) their holdings change inducing an extra common factor on asset prices.[29] This is the source of excess correlation.

[28] This "specialization" is not necessarily assumed, but rather could be proven to be an equilibrium emanating from the information acquisition stage.
[29] Extra in a sense that it is in addition to the assumed common factors on the fundamentals.

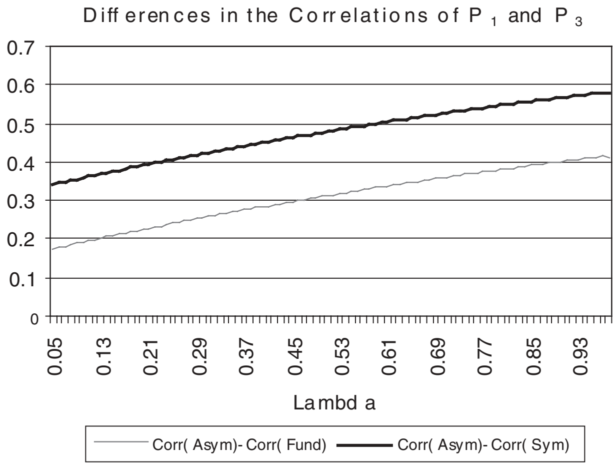

Figure 1. Excess Correlation I: Comparing price correlation varying the measure of agents informed about each random variable. We have made use of the definition: $Corr(Fund) = Corr(X_1, X_3)$

This phenomenon does not happen in the perfect information case since all agents trade equally in all assets, there is no specialization.[30]

As expressed repeatedly, all results were in the expected direction, endorsing our view that what is known as contagion can be explained theoretically by an asymmetric information model. One remark is in place. For the results shown here we have fixed $\lambda = 1/2$. It should be clarified that several other values of λ were used and the same exercises were done. The results do not depend on λ at all. So, independently of the outcome of a potential information acquisition stage all results reported hold. More importantly, it can be shown that this value for λ constitute an equilibrium. Figure 1 below shows the effects of varying λ on the differences of correlation. Results are as expected: as we increase λ we are increasing the percentage of traders that have information about assets 1 and 3 and prices will reflect more and more their information. So, as λ increases P_1 and P_3 reflect more and more I_1, and, given that I_1 is common to both, the excess correlation phenomenon is exacerbated.

An interesting exercise is to see what happens when we let the variance of the assets' supply (noise trading variance) change. More precisely, we let the noise trading variance go to zero to see what happens to the correlations. As this variance is reduced, the residual variance in prices is explained mainly by the information variables. Hence, as noise trading variance goes to zero, since we have common information variables, correlation in the symmetric and asymmetric case increases - price correlation is decreasing in the volatility of noise. However, one would expect that as the noise in markets is reduced the importance of asymmetries would be reduced. As the noise is reduced, prices become more informative and private

[30] A similar phenomenon is sometimes termed "The Habitat View of Co-movement" (e.g. [7] and [8]) since it relies on investors having a preferred habitat. Here these investors can rationally choose to have a preferred habitat, it needs not be an assumption, and it can be the outcome of an information acquisition stage.

signals less valuable, therefore the difference between the symmetric and asymmetric cases is reduced. If our intuition is correct, as soon as the variance gets small enough we should get a partial reversion of our results.

Figure 2 presents one experiment in these lines. As we can see, since the correlation in fundamentals is independent of noise trading and the correlation in the asymmetric case is decreasing in the variance of noise trading, the difference of these two increases as the variance approaches zero. On the other hand, as expected, the difference in the asymmetric and symmetric case is reduced, even becoming negative, indicating that the correlation in the symmetric case increases faster than in the asymmetric one, as noise trading variance goes to zero. We also run a modified simulation, not presented here, by letting the noise trading variance in markets

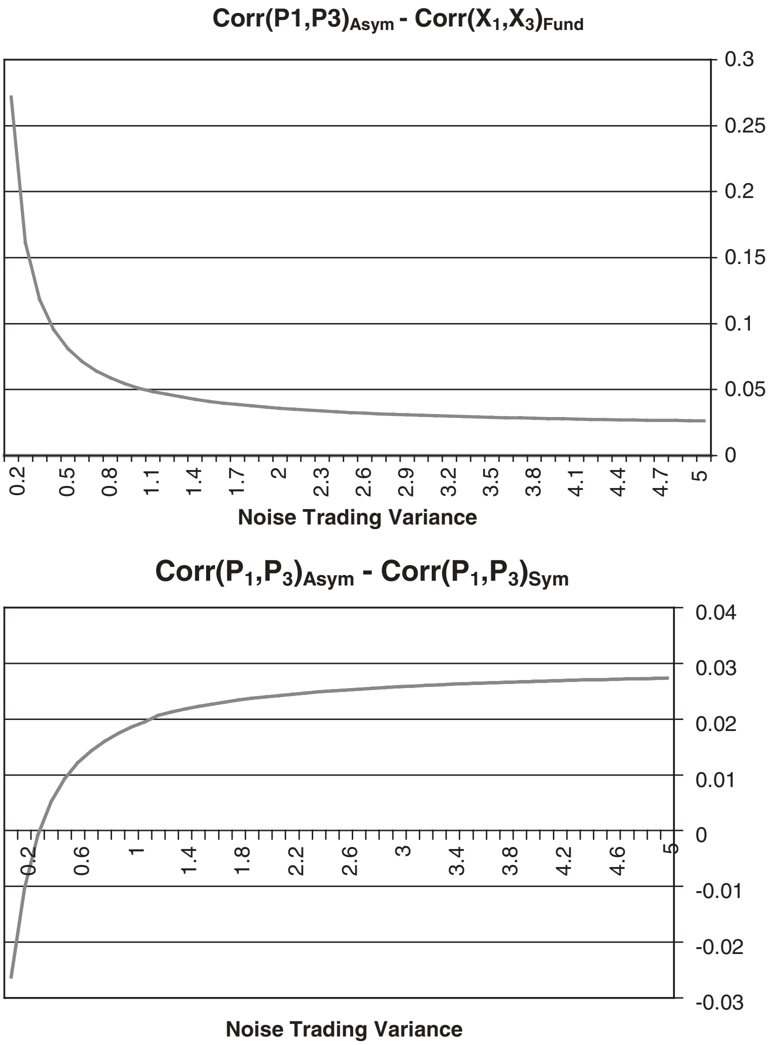

Figure 2. Excess Correlation II: Comparing price correlation varying the noise trading variance

1 and 3 go to zero, holding constant the parameters in market 2. As expected the results are similar to the ones reported above since what matters is the importance of the information vis-à-vis noise in the prices of assets 1 and 3, and this is mostly unaffected by the parameters of market 2. It is important to notice that the reversion of sign does not necessarily happen, only for some parameter values. The reversion happens mainly for low ϖ when asymmetries are unimportant; whenever asymmetric information really matters the effect of reduced noise trading is not enough to wash out the effects.

One more numerical analysis developed interesting results, this analysis came to mind as an answer to the question: Can we think about the symmetric equilibrium as a limit of the asymmetric one? The answer is yes. By letting the volatility of the information variables go to zero we should approximate the symmetric equilibrium model. Why? Because, as the information variables become deterministic, they end up in every agent's information set, hence we are back to the full information symmetric equilibrium. To try and verify this conjecture we let $\varpi \to 0$, and compare the resulting parameter values with the ones for the full info symmetric case. As $\varpi \to 0$, we get that the relevant matrices of coefficients in the asymmetric case converge to the same structure as in the symmetric case, i.e. the limit of the asymmetric case has matrices that have zero entries where they should and all the signs are as needed.

We also verify that as $\varpi \to 0$ the excess correlation is reduced, and it increases with ϖ. This means that as information becomes more volatile the excess correlation phenomenon is exacerbated. In the Introduction it was said that this model is relevant to explain financial contagion since during crises informational asymmetries are more important. The previous exercise sheds light into this justification. Crises are usually accompanied by increased noise in financial indicators and fundamentals, here an increase in ϖ can be interpreted as an increase in this noise. During crises the informational (fundamental) part of assets' payoffs becomes more noisy, and, as shown here, this exacerbates the financial contagion phenomenon. So, this model explains why excess correlation is observed mostly during crises, since in calmer periods information is less volatile ($\varpi \to 0$).

Recently, end of 2001 beginning of 2002, Argentina faced a severe crisis emanating, mostly, from its fixed exchange rate regime. This crisis, however, did not contaminate any neighboring markets, at least not much. Two questions come to mind: Why was this the case? Can the present model explain this instance of immunity?

Casual observations[31] seem to suggest that there are some characteristics of this crisis in Argentina that fit the current model and might help explain the mentioned immunity of neighboring economies. It seems that the problem in Argentina has been easily pinpointed as a result of the fixed exchange regime and its currency board. This institutional characteristic is not shared by Argentina's neighbors; hence the problems seem very localized. Also, in general, these were not times of extremely volatile information concerning developing markets. The shock seems to be of an idiosyncratic nature. So, it would not be farfetched to associate this crisis in Argentina with a situation where ϖ is pretty low and σ is high. This interpretation

[31] I would like to thank Professors at Princeton, macroeconomic consultants in Brazil and friends working in the financial markets for private conversations on this subject.

would imply, by the comparative statics exercises reported above, that the excess correlation phenomenon would be very much attenuated. Hence, if this interpretation is accepted, we see that, within the proposed model, one would not observe a lot of contagion from Argentina to neighboring markets, which is, indeed, verified empirically.

As mentioned before, several empirical studies analyze the existence of asymmetric information in international markets and its importance during crises; in general the evidence points to the existence of asymmetries in favor of "nationals". For example, German traders seem to know more about German stocks than other European traders. This evidence reinforces our view of informational asymmetries - an asymmetry in terms of what you know and not whether or not you know anything. The latter interpretation is the one in [29] and, as discussed in the Related Literature section, has no substantiation in the empirical literature.

6.1.2 Comparing return correlations

In this section we try to reinforce the validity of our results. So far the main focus of the analysis was the comparison of price correlations in two different equilibria and the comparison of price and payoff correlation. Focus now is shifted to returns. Is there excess correlation in returns? The random variables that will be analyzed now are $X_i - P_i$, $i \in \{1, 3\}$ and we are interested in:

$$\Delta \stackrel{d}{=} Corr_{Asym.\ Eqm}(X_1 - P_1, X_3 - P_3) - Corr_{Sym.\ Eqm}(X_1 - P_1, X_3 - P_3). \tag{16}$$

A lot of numerical exercises were performed always delivering the desired result that $\Delta > 0$. Actually, even though before we had found an instance in which $D_2 < 0$ (see Figure 2), this new exercise did not deliver $\Delta < 0$ for any parameter tested. The same comparative statics were done, and the same qualitative results were obtained.

6.2 Analysis of home-bias effect

The fact that capital flows across countries can be explained by shared informational variables can be seen as a “modified home-bias”. [32] documents this empirically and shows that, the closer, culturally and geographically, two countries are, the higher is the capital flows between them. We use our model to justify this phenomenon theoretically. We show that a trader informed about asset 1 (and therefore 3) trades, in equilibrium, more heavily on assets 1 and 3 than on asset 2. Therefore, if we think of a trader that have information about asset 1 as someone living in country 1, we are showing that residents of country 1 exchange more capital with country 3 than with country 2. And, at the same time, retain more of their funds at home than at country 2. Note that the aggregate trading on country 1 and 3, for this individual, is higher than his trading on country 2: He is trading more in his “home-region” than in foreign regions.

Recall that the optimal demand for a trader informed about asset one is given by $\frac{1}{\rho}(V_1)^{-1}[E_1 - PR]$. So, for a given price and conditional expected value (take

$[E_1 - PR] = c$), the "aggressiveness" of the portfolio can be represented by $(V_1)^{-1}$. We analyze this matrix and show that the sum of the elements of its first row (which is equal to the sum of the elements of the third row) is bigger than that of the second row. Hence, the aggressiveness of this portfolio in assets 1 and 3 is higher than that in asset 2. In the proof to Proposition 2 we show that

$$(V_1)^{-1} = \begin{bmatrix} \frac{1}{\sigma^2} & 0 & 0 \\ 0 & \frac{1}{A+\sigma^2} & 0 \\ 0 & 0 & \frac{1}{\sigma^2} \end{bmatrix}, \tag{17}$$

with $A = \frac{g_{22}^2 v_2 \varpi}{g_{22}^2 v_2 + b_{22}^2 \varpi}$. Since one can easily see that A is greater than zero, it is immediate that the trading intensity for a Type 1 investor on asset 1 (and 3) is higher than his intensity on asset 2. One could do the very same exercise for $(V_2)^{-1}$ to show that Type 2 investors trade more aggressively on asset 2 than on asset 1 or 3.

Taking $[E_1 - PR] = c > 0$ we find numerically that investors might trade up to 67% more of their portfolio in home assets. In this simple framework, where trades and holdings are inherently intertwined, this means that their portfolio consists of 83.5% in home stocks (assets 1 and 3) and 16.5% in international stocks (asset 2). This is in line with [18]. However, if $c < 0$ he will also be short selling more of the home assets. So, this model is more appropriate to explain excess flows between similar countries and not holdings.

Our intuition and motivation for this model as a potential theoretical justification of [32]'s empirical findings is verified analytically. Agents do trade more aggressively on assets that they are familiar with. So, for example, someone living in Brazil (or a brokerage firm located there) that has information about the Brazilian and Argentine assets will trade more heavily on them. The empirical findings that culture and information sharing matters a lot to explain capital flows between countries has, in this model, a sound theoretical counterpart.

It should be mentioned that other papers also have an explanation based on information asymmetries for the phenomenon of home-bias; e.g. [19] and [33]. Both these papers analyze the traditional home-bias effect, and not this modified one that deals with flows. They do not address financial contagion and do not justify the appearance of the information asymmetries, they do not endogenize the information acquisition. Also, it can be shown that [19]'s model cannot have contagion from one asset to another. He deals with 2 assets and, with the structure he imposes in his model, it can be shown that prices are uncorrelated, even in the asymmetric information case.

7 Imperfectly competitive market structure

In this section we change the assumptions concerning the market microstructure. So far we dealt with competitive markets and "limit orders" (whole demand schedules). To investigate the robustness of our results we allow a different market setting. We assume that markets are imperfectly competitive and only market orders are allowed, closely following [30] in the approach. Our theoretical results are different, though, as we deal with multiple insiders, multiple assets and information asymmetries. [10] generalized [30]'s model to allow for multiple assets and multiple insiders. We diverge considerably from their assumptions about the information structure. First, in their assumption A3 they restrict the informational advantage of each insider to have the same precision.[32,33] In our approach we allow traders to have signals of different precision . They also require that the variance of the informational advantage be non-singular. This requires that each trader has a non-degenerate signal for each asset, i.e. each trader receive some information about each asset's payoff.[34] In our asymmetric information model each insider has information about only one asset and, hence, has an informational advantage with a singular variance matrix . Therefore, our asymmetric model has a theoretical value of its own.

The model has 3 types of insiders: Each informed about only one of the assets. Assets 1 and 2 have, without loss of generality, positively correlated values, while asset 3 is independent of both. There are also noise traders that give the insiders a chance to "hide" their information from the market maker. Trades take place in period 1 and the value of the asset realizes in period 2, when consumption takes place. The objective is to compare this situation with one in which an insider is informed about all the assets. We show that the correlation in the asymmetric information case is higher than in the symmetric one. The correlation in the perfect information case is always smaller than the correlation in fundamentals, and the correlation in the asymmetric information case is higher than the fundamental correlation. [35]

Risk-neutral insiders have information concerning the value of the assets. Uninformed individuals behave like noise traders. And a market maker sets price upon observing the aggregate order flow for each asset separately. When we deal with the asymmetric information case, we assume that each insider cannot observe the other insider's demand.[36] The ex-post liquidation value of the assets, X, and the noise trader's demand, U, are independent normally distributed random variables with mean zero and variance Σ and V, respectively, where:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}, \quad V = \begin{bmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & v \end{bmatrix}.$$

[32] See [10], p. 696.

[33] In their notation s_k is the signal of trader k, v is the assets value, and they assume that:

$$\xi_k \equiv E\left[v|s_k\right] - E\left[v\right] \sim N(0, \Sigma_\xi)$$

so the variance of each trader's informational advantage is the same. Hence, traders receive signals of the same precision.

[34] In the notation of the previous footnote they require Σ_ξ to be non-singular.

[35] Of course, for ex-ante correlations that approach one, these differences basically disappear.

[36] This assumption seems to be justified since if you submit an order to your broker you do not expect anybody, other than the broker himself, to know your order.

The assumption of independence of the noise trading across markets could be relaxed, but it would not add to the insight of the model.[37] Without loss of generality we assume $\sigma_{12} > 0$.

Our definition of equilibrium is the same as in [30], and we omit it for sake of brevity. (It entails profit maximization on the part of traders and semi-strong market efficiency on the part of the market maker.) First, we characterize the equilibrium in the general case of N insiders each one with perfect information:[38,39]

Proposition 3. *A LSE is given by:*

$$D_i(X) = \beta_{N,pi} \cdot X,\ P\left(\sum_{i=1}^{N} D_i + U\right) = \lambda_{N,pi} \cdot \left(\sum_{i=1}^{N} D_i + U\right), \forall i = 1, ..., N \tag{18}$$

where $\beta_{N,pi}$ and $\lambda_{N,pi}$ are 3 by 3 matrices (and the subscript N, pi stands for perfect information with N insiders) with

$$\lambda_{N,pi} = \frac{\sqrt{N}}{(N+1)\sqrt{v}} \Sigma^{1/2}; \quad \beta_{N,pi} = \sqrt{\frac{v}{N}} \Sigma^{-1/2}.$$

Now, we derive the equilibrium in the asymmetric information case. We assume that there are 3 insiders, each informed about one asset. We make the additional assumption that each insider cannot observe the other's market order. We compare this asymmetric information case with the perfect information case (for $N = 1, 2, 3$).

Proposition 4. *The equilibrium is characterized by β_{ai}^i (3 by 1 vectors) and λ_{ai} (3 by 3 matrix), where ai stands for asymmetric information with 3 insiders and*

$$\beta_{ai}^1 = \frac{1}{2}\lambda_{ai}^{-1} \begin{pmatrix} 1 \\ \frac{\sigma_{12}}{\sigma_1^2} \\ 0 \end{pmatrix}, \quad \beta_{ai}^2 = \frac{1}{2}\lambda_{ai}^{-1} \begin{pmatrix} \frac{\sigma_{12}}{\sigma_2^2} \\ 1 \\ 0 \end{pmatrix}, \quad \beta_{ai}^3 = \frac{1}{2}\lambda_{ai}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

and are fully determined by

$$\lambda_{ai} = \frac{1}{2\sqrt{v}} \left[\begin{pmatrix} 3\sigma_1^2 + \frac{5}{\sigma_2^2}\sigma_{12}^2 & 7\sigma_{12} + \frac{1}{\sigma_1^2\sigma_2^2}\sigma_{12}^3 & 0 \\ 7\sigma_{12} + \frac{1}{\sigma_1^2\sigma_2^2}\sigma_{12}^3 & 3\sigma_2^2 + \frac{5}{\sigma_1^2}\sigma_{12}^2 & 0 \\ 0 & 0 & 3\sigma_3^2 \end{pmatrix} \right]^{\frac{1}{2}}, \tag{19}$$

[37] In a previous version of this paper we run a brief analysis of the results for correlated noise trading and they were as desired.

[38] We could make the additional assumption that each insider cannot observe the other's order. However, since both know the vector of ex-post liquidation values, X, this is immaterial in a symmetric equilibrium. By knowing X and its own matrix of coefficients, the insider can perfectly infer the other's demand. This assumption will be necessary in the asymmetric information case, though.

[39] The proof is omitted to save space.

We proceed with a numerical analysis, concentrating on the price correlation of assets 1 and 2. The numerical analysis performed shows that our intuition is indeed correct. For all the parameter values used to solve the model (and $N=1,2,3$) the correlation between the prices of assets 1 and 2 is higher in the asymmetric information case than in the perfect/symmetric information one. For values of the ex-ante correlation below 1, the correlation in the asymmetric case is also higher than the former. To interpret our results, first notice that the solution for the perfect information leads to a vector of market orders by insiders that is directly proportional to

$$\begin{bmatrix} \frac{\sigma_2^2}{\sigma_1^2\sigma_2^2-\sigma_{12}^2} & \frac{-\sigma_{12}}{\sigma_1^2\sigma_2^2-\sigma_{12}^2} & 0 \\ \frac{-\sigma_{12}}{\sigma_1^2\sigma_2^2-\sigma_{12}^2} & \frac{\sigma_1^2}{\sigma_1^2\sigma_2^2-\sigma_{12}^2} & 0 \\ 0 & 0 & \frac{1}{\sigma_3^2} \end{bmatrix}^{\frac{1}{2}} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}.$$

So, a good signal about asset 2, all else equal, reduces the insider's trading on asset 1, and vice-versa. Intuitively this happens because he needs to balance how much information about asset 2 is revealed through his trading in asset 1. If he has good information about asset 2 that needs to be "hidden", he will trade less in asset 1, so that less info about asset 2 gets incorporated into prices through this "indirect" channel. He "uses" his information advantage by trading directly more on asset 2, and less on 1. Therefore, the ex-ante correlation on assets' payoffs is played down on the equilibrium price. In the asymmetric equilibrium each insider only has information about one asset, hence this phenomenon does not happen. For instance, insider 2 only has information about the second asset, hence he might as well use it to trade in asset 1 - he does not have "direct" information about asset 1 so cannot expect to trade on it using such information. If he trades less on asset 1 to hide his information, he just leaves money on the table. Essentially, what happens is that insider 2 views assets 1 and 2 as a composite portfolio that depends (with different weights) on the information he has about asset 2. While in the symmetric equilibrium insiders may downplay the indirect information and use more aggressively the direct one. In summary, in the symmetric equilibrium, with all else fixed, the better the information about asset 2 is, the smaller the trading on asset 1 will be, in the asymmetric case the better the information about asset 2 is, the bigger the trading on asset 1 will be. Hence, the exacerbated correlation. Changing the parameters of the model has very little effect on the results.

8 Concluing remarks

The results in this paper strengthen our conjecture of financial contagion as being generated by asymmetric information in financial markets. We are able to order correlation among assets sharing a common factor (assets 1 and 3), for times of crises: Correlation in the asymmetric equilibrium was higher than in payoffs, which was higher than in the symmetric equilibrium. And, return correlation was higher under information asymmetries. The shear presence of the markets, without any imperfection or asymmetry helps diversify away risk reducing the correlation of asset prices. However, as soon as we introduce information asymmetries, maintaining markets frictionless, we obtain excess correlation. The fact that the results do not depend on the market microstructure further reinforces our intuition.

We also provide theoretical support for the empirical work of [32]. They find that capital flows are closely linked with geographical proximity, that serves as a proxy for shared cultural/informational ties. Here, we show that this phenomenon (that we term "modified home-bias") might be the equilibrium outcome of a model of asymmetric information in financial markets.

Additionally,we provide closed form solutions for a model with multiple insiders and assets that generalize the ideas in [10] by allowing for information asymmetries across insiders and not only between insiders and noise traders.

A Omitted proofs

A.1 Proof of Proposition 1

The market clearing condition is: $D^*(I_1, I_2) = U$, or, plugging in the functional forms: $(\rho\Sigma)^{-1}(I - PR) = U$. Solving for P we have:

$$P = \frac{1}{R} I - \frac{\rho}{R} \Sigma U = \begin{bmatrix} \frac{1}{R} & 0 \\ 0 & \frac{1}{R} \\ \frac{1}{R} & 0 \end{bmatrix} * \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - \begin{bmatrix} \frac{\rho\sigma^2}{R} & 0 & 0 \\ 0 & \frac{\rho\sigma^2}{R} & 0 \\ 0 & 0 & \frac{\rho\sigma^2}{R} \end{bmatrix} * \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}. \tag{20}$$

Therefore by equating coefficients we get result stated in the proposition.

A.2 Proof of Proposition 2

Let us first concentrate in setting up the general system of equations that will be used to run the numerical exercises, after that we tackle the issues of existence and try to elaborate on the format of the solution. Before we start, one can easily see that α must be a vector of zeroes, as all random variables have zero mean. The market clearing equation can be written as

$$\frac{\lambda}{\rho}(V_1)^{-1}[E_1 - PR] + \frac{1-\lambda}{\rho}(V_2)^{-1}[E_2 - PR] = U. \tag{21}$$

Note that E_i and V_i depend on P as well, so we need to open these factors. Notice that

$$V_1 = \Omega_2 + \Sigma - g_1 j_1^{-1} g_1^T \text{ and } V_2 = \Omega_2 + \Sigma - g_2 j_2^{-1} g_2^T$$

where the matrices g_i and j_i are given by

$$g_1 = \left(\begin{pmatrix} \Omega \\ (\varpi\ 0) \end{pmatrix} \beta^T \begin{pmatrix} \varpi \\ 0 \\ \varpi \end{pmatrix} \right); \;\; g_2 = \left(\begin{pmatrix} \Omega \\ (\varpi\ 0) \end{pmatrix} \beta^T \begin{pmatrix} 0 \\ \varpi \\ 0 \end{pmatrix} \right);$$
$$j_1 = \begin{pmatrix} \beta\Omega\beta^T & \beta \begin{pmatrix} \varpi \\ 0 \end{pmatrix} \\ (\varpi\ 0)\, \beta^T & \varpi \end{pmatrix}; \;\; j_2 = \begin{pmatrix} \beta\Omega\beta^T & \beta \begin{pmatrix} 0 \\ \varpi \end{pmatrix} \\ (0\ \varpi)\, \beta^T & \varpi \end{pmatrix}.$$

And,

$$j_1^{-1} = \begin{pmatrix} a_1^{-1} & -\left[(1\ 0)\, \beta^T a_1^{-1} \right]^T \\ -\left[(1\ 0)\, \beta^T a_1^{-1} \right] & y_1^{-1} \end{pmatrix},$$
$$j_2^{-1} = \begin{pmatrix} a_2^{-1} & -\left[(0\ 1)\, \beta^T a_2^{-1} \right]^T \\ -\left[(0\ 1)\, \beta^T a_2^{-1} \right] & y_2^{-1} \end{pmatrix}$$

where

$$a_1^{-1} = \left\{ \beta\Omega\beta^T + \gamma V \gamma^T - \beta \begin{pmatrix} \varpi \\ 0 \end{pmatrix} \frac{1}{\varpi} (\varpi\ 0)\, \beta^T \right\}^{-1};$$
$$a_2^{-1} = \left\{ \beta\Omega\beta^T + \gamma V \gamma^T - \beta \begin{pmatrix} 0 \\ \varpi \end{pmatrix} \frac{1}{\varpi} (0\ \varpi)\, \beta^T \right\}^{-1};$$
$$y_1^{-1} = \left\{ \varpi - (\varpi\ 0)\, \beta^T \left[\beta\Omega\beta^T + \gamma V \gamma^T \right]^{-1} \beta \begin{pmatrix} \varpi \\ 0 \end{pmatrix} \right\}^{-1};$$
$$y_2^{-1} = \left\{ \varpi - (0\ \varpi)\, \beta^T \left[\beta\Omega\beta^T + \gamma V \gamma^T \right]^{-1} \beta \begin{pmatrix} 0 \\ \varpi \end{pmatrix} \right\}^{-1}.$$

Finally, we can also write

$$E_i = g_i j_i^{-1} \begin{pmatrix} P \\ I_i \end{pmatrix}$$

and we can calculate $g_i j_i^{-1} = (x_i\ z_i)$, where

$$x_1 = \left[\begin{pmatrix} \Omega \\ (\varpi\ 0) \end{pmatrix} + \begin{pmatrix} \varpi \\ 0 \\ \varpi \end{pmatrix} (-1\ 0) \right] \beta^T a_1^{-1};$$
$$x_2 = \left[\begin{pmatrix} \Omega \\ (\varpi\ 0) \end{pmatrix} + \begin{pmatrix} 0 \\ \varpi \\ 0 \end{pmatrix} (0\ -1) \right] \beta^T a_2^{-1};$$
$$z_1 = \begin{pmatrix} \Omega \\ (\varpi\ 0) \end{pmatrix} \beta^T \left[(-1\ 0)\, \beta^T a_1^{-1} \right]^T + \begin{pmatrix} \varpi \\ 0 \\ \varpi \end{pmatrix} y_1^{-1};$$

$$z_2 = \begin{pmatrix} \Omega \\ (\varpi\ 0) \end{pmatrix} \beta^T \left[\begin{pmatrix} -1\ 0 \end{pmatrix} \beta^T a_2^{-1} \right]^T + \begin{pmatrix} 0 \\ \varpi \\ 0 \end{pmatrix} y_2^{-1}.$$

So we can re-write $E_i = x_i P + z_i I_i$. Solving (21) for P we get:

$$P = \frac{V_1}{R\lambda} \left(\frac{V_1}{\lambda} + \frac{V_2}{(1-\lambda)} \right)^{-1} \frac{V_2}{(1-\lambda)} \left[\lambda V_1^{-1} E_1 + (1-\lambda) V_2^{-1} E_2 - \rho U \right], \quad (22)$$

and if we define

$$T \stackrel{d}{=} \frac{V_1}{R\lambda} \left(\frac{V_1}{\lambda} + \frac{V_2}{(1-\lambda)} \right)^{-1} \text{ and } X \stackrel{d}{=} T \frac{\lambda}{(1-\lambda)} V_2 V_1^{-1}, \quad (23)$$

we have, after tedious calculations that the price vector may be written as:

$$\begin{aligned} P &= X E_1 + T E_2 - \frac{\rho}{1-\lambda} T V_2 U = X\left(x_1 P + z_1 I_1\right) + T\left(x_2 P + z_2 I_2\right) \\ &\quad - \frac{\rho}{1-\lambda} T V_2 U \qquad (24) \\ &= N^{-1} \left[X z_1 I_1 + T z_2 I_2 - \frac{T}{(1-\lambda)} V_2 \rho U \right]; \qquad (25) \end{aligned}$$

with $N^{-1} = [I_D - X x_1 - T x_2]^{-1}$, where I_D is the 3×3 identity matrix. Equating coefficients with the conjectured price function we get the desired non-linear system.

Now, let us turn to the issue of existence. In order to do that we first notice that our conjectured price function should satisfy

$$\alpha = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \beta = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \\ b_1 & 0 \end{bmatrix} \text{ and } \gamma = \begin{bmatrix} g_1 & 0 & g_3 \\ 0 & g_2 & 0 \\ g_3 & 0 & g_1 \end{bmatrix}, \quad (26)$$

because X_1 and X_3 are identically distributed and X_2 is independent of all variables for assets 1 and 3. So, P_1 and P_3 do not depend on I_2, P_2 does not depend on I_1, the effect of I_1 on P_1 and P_3 is the same, the supply of asset 1 affects its price in the same way that the supply of asset 3 affects its own price ($g_{11} = g_{33} \equiv g_1$), the supply of asset 3 affects the price of asset 1 in the same way that the supply of asset 1 affects the price of asset 3 ($g_{13} = g_{31} \equiv g_3$) and the supply of asset 2 does not affect markets 1 and 3, and vice-versa ($g_{12} = g_{21} = g_{23} = g_{32} = 0$ and $g_{22} \equiv g_2$). Given this structure one can easily notice that

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} b_1 I_1 - g_1 U_1 - g_3 U_3 \\ b_2 I_2 - g_2 U_2 \\ b_1 I_1 - g_3 U_1 - g_1 U_3 \end{pmatrix}$$

and

$$E_1 = \begin{pmatrix} I_1 \\ E\left[X_2 | P_2\right] \\ I_1 \end{pmatrix}, \; E_2 = \begin{pmatrix} E\left[X_1 | P_1, P_3\right] \\ I_2 \\ E\left[X_3 | P_1, P_3\right] \end{pmatrix},$$

with

$$E\left[X_2|P_2\right]=\frac{b_2\varpi}{b_2^2\varpi+g_2^2v_2}P_2$$

and

$$E\left[X_1|P_1,P_3\right]=E\left[X_3|P_1,P_3\right]=\left(\,b_1\varpi\;b_1\varpi\,\right)A^{-1}\begin{pmatrix}P_1\\P_3\end{pmatrix}$$

with

$$A^{-1}=\frac{1}{D}\begin{pmatrix}b_1^2\varpi+\left(g_1^2+g_3^2\right)v_1 & -b_1\varpi-2g_1g_3v_1\\ -b_1\varpi-2g_1g_3v_1 & b_1^2\varpi+\left(g_1^2+g_3^2\right)v_1\end{pmatrix};$$

$$D=\left[b_1^2\varpi+\left(g_1^2+g_3^2\right)v_1\right]^2-\left[b_1\varpi+2g_1g_3v_1\right]^2.$$

Conditional variances can also be calculated as

$$V_1=Var\left(X|I_1,P_2\right)=\begin{pmatrix}\sigma^2 & 0 & 0\\ 0 & m_1 & 0\\ 0 & 0 & \sigma^2\end{pmatrix};\;V_1^{-1}=\begin{pmatrix}\frac{1}{\sigma^2} & 0 & 0\\ 0 & \frac{1}{m_1} & 0\\ 0 & 0 & \frac{1}{\sigma^2}\end{pmatrix}$$

$$m_1=Var\left(X_2|P_2\right)=\varpi+\sigma^2-\frac{b_2^2\varpi^2}{b_2^2\varpi+g_2^2v_2},$$

And,

$$V_2=Var\left(X|I_2,P_1,P_3\right)=\begin{pmatrix}A & 0 & B\\ 0 & \sigma^2 & 0\\ B & 0 & A\end{pmatrix};$$

notice that

$$\begin{pmatrix}A & B\\ B & A\end{pmatrix}=Var\left(\begin{pmatrix}X_1\\X_3\end{pmatrix}|P_1,P_3\right)=\begin{pmatrix}\sigma^2+\varpi & \varpi\\ \varpi & \sigma^2+\varpi\end{pmatrix}-KA^{-1}K^T$$

$$K=\begin{pmatrix}b_1\varpi & b_1\varpi\\ b_1\varpi & b_1\varpi\end{pmatrix};$$

therefore

$$V_2=\begin{pmatrix}\sigma^2+\varpi-L & 0 & \varpi-L\\ 0 & \sigma^2 & 0\\ \varpi-L & 0 & \sigma^2+\varpi-L\end{pmatrix};$$

$$L=\frac{2b_1^2\varpi^2\left[b_1\varpi\left(b_1-1\right)+v_1\left(g_1-g_3\right)^2\right]}{\left[b_1^2\varpi+\left(g_1^2+g_3^2\right)v_1\right]^2-\left[b_1\varpi+2g_1g_3v_1\right]^2},$$

and

$$V_2^{-1}=\begin{pmatrix}\frac{\sigma^2+\varpi-L}{d} & 0 & \frac{\varpi-L}{d}\\ 0 & \frac{1}{\sigma^2} & 0\\ \frac{\varpi-L}{d} & 0 & \frac{\sigma^2+\varpi-L}{d}\end{pmatrix};\;d=\left(\sigma^2+\varpi-L\right)^2-\left(\varpi-L\right)^2.$$

Now, let us use this derived structure to re-think the problem starting from the market clearing condition (21) now re-written as

$$\lambda \begin{pmatrix} \frac{1}{\sigma^2} & 0 & 0 \\ 0 & \frac{1}{m_1} & 0 \\ 0 & 0 & \frac{1}{\sigma^2} \end{pmatrix} \begin{pmatrix} I_1 - P_1 R \\ \frac{b_2 \varpi}{b_2^2 \varpi + g_2^2 v_2} P_2 - P_2 R \\ I_1 - P_3 R \end{pmatrix} +$$

$$(1-\lambda) \begin{pmatrix} \frac{\sigma^2+\varpi-L}{d} & 0 & \frac{\varpi-L}{d} \\ 0 & \frac{1}{\sigma^2} & 0 \\ \frac{\varpi-L}{d} & 0 & \frac{\sigma^2+\varpi-L}{d} \end{pmatrix} \begin{pmatrix} \left(b_1\varpi \; b_1\varpi \right) A^{-1} \begin{pmatrix} P_1 \\ P_3 \end{pmatrix} - P_1 R \\ I_2 - P_2 R \\ \left(b_1\varpi \; b_1\varpi \right) A^{-1} \begin{pmatrix} P_1 \\ P_3 \end{pmatrix} - P_3 R \end{pmatrix} = \rho U$$

Since L, A and d only depend on market 1 and 3 parameters and m_1 only depend on market 2 parameters and, given the structure of the variance matrices, we can now see that this problem splits into two separate ones: One for asset 2 and the other for assets 1 and 3 together.

The first problem is much like [24], i.e., a market with a proportion (here $1-\lambda$) of agents informed about asset 2 and the remainder uninformed. Existence and uniqueness (in the class of linear equilibria) was demonstrated there and we refer the interested reader to the source. The second problem, for assets 1 and 3 together, is just a 2-dimensional version of [24] (now with a proportion λ being informed) and a similar method to prove existence and uniqueness can be used (just substituting scalars by matrices the proof follows almost literally). The interesting part of this proof was the characterization of the equations that determine the equilibrium and that are later used to numerically illustrate the contagion phenomenon.

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